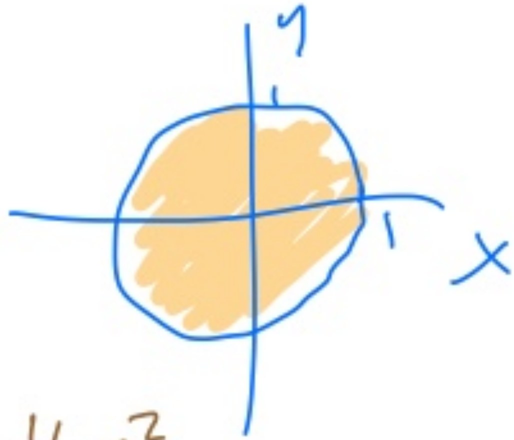
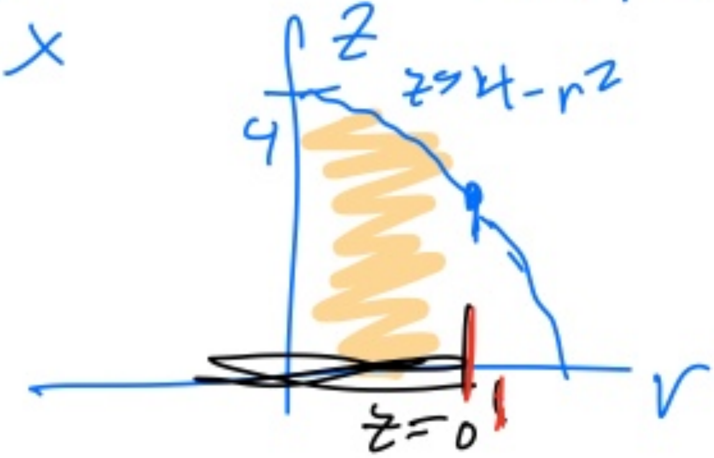
3.30 Avg value of $\overset{F(x,y,z)}{2x^2+3x^2z+2024y}$

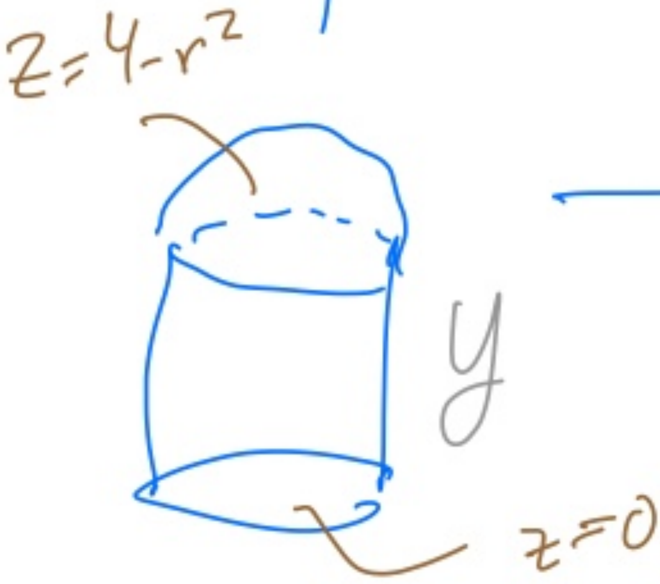
over $\mathcal{Y} = \{(x,y,z) : x^2+y^2 \le 1,\ 0 \le z \le 4-x^2-y^2\}$.



$z=0$ to $z=4-x^2-y^2$

$z=4-r^2$





$$\begin{pmatrix}\text{Avg value of} \\ F(x,y,z) \\ \text{on } \mathcal{Y}\end{pmatrix} = \frac{\iiint_{\mathcal{Y}} F(x,y,z)\,dx\,dy\,dz}{\iiint_{\mathcal{Y}} 1\,dx\,dy\,dz}$$

$$\text{Top} = \int_{\theta=0}^{2\pi}\int_{r=0}^{1}\int_{z=0}^{4-r^2} (F)\, r\,dz\,dr\,d\theta$$

$x = r\cos\theta,\ y = r\sin\theta,\ z = z$

Graph these vector fields.

(1) $V(x,y) = (x,y)$

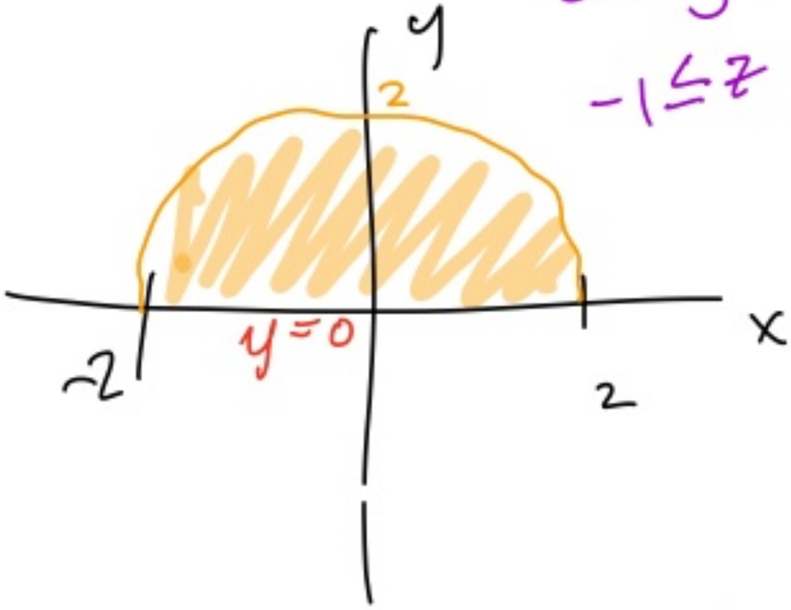
(2) $V(x,y) = (-y,x)$

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{-1}^{1} \tau \, dz\, dy\, dx$$

$-2 \le x \le 2$

$0 \le y \le \sqrt{4-x^2}$

$-1 \le z \le 1$

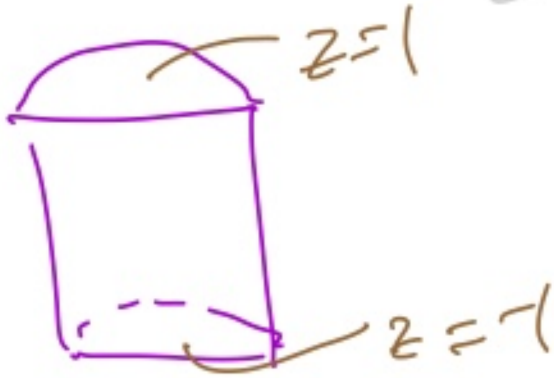


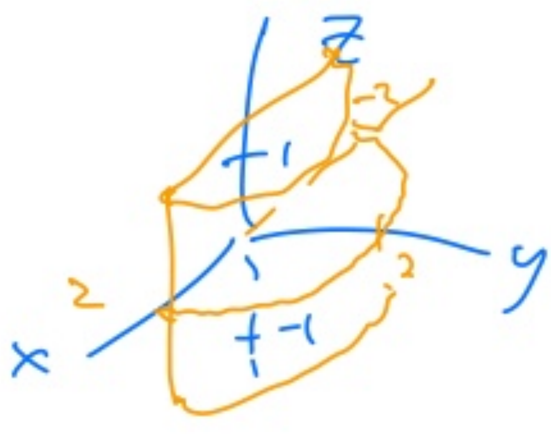
$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$

$z = -1 \rightarrow z = 1$





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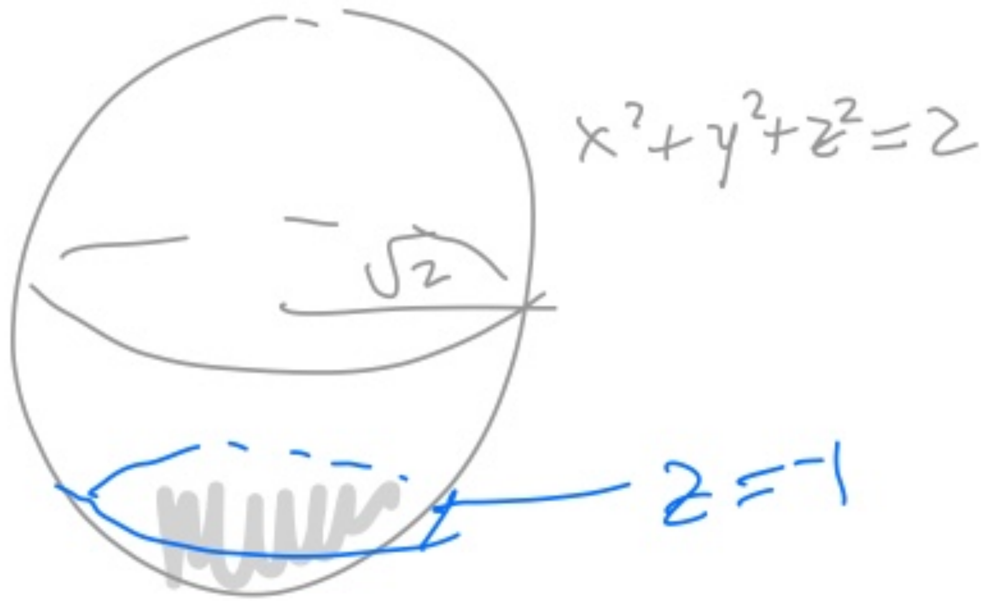
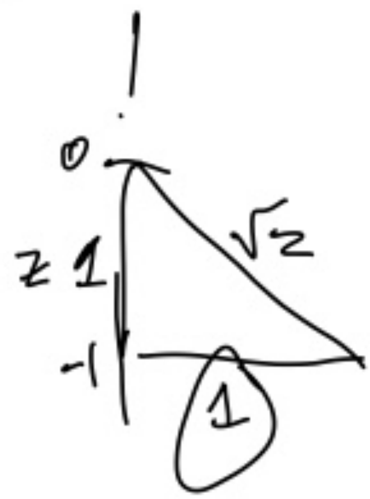
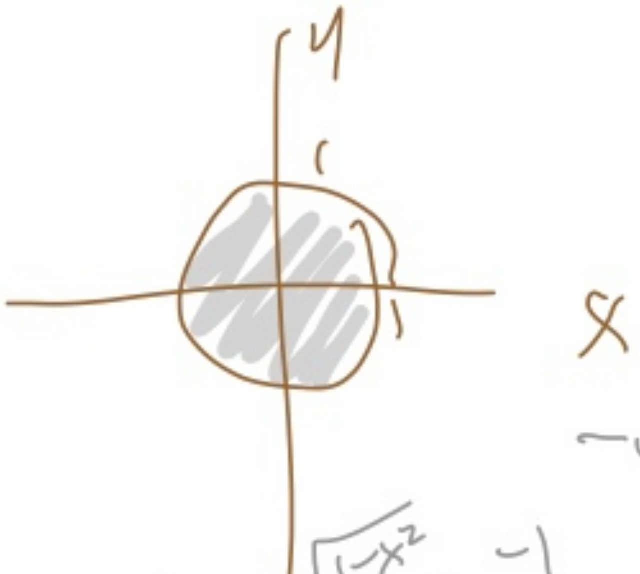






$$-\sqrt{2-x^2-y^2} \le z \le -1$$

$$\int_{-1}^{1}\int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{z=-\sqrt{2-x^2-y^2}}^{-1} dz\,dy\,dx$$

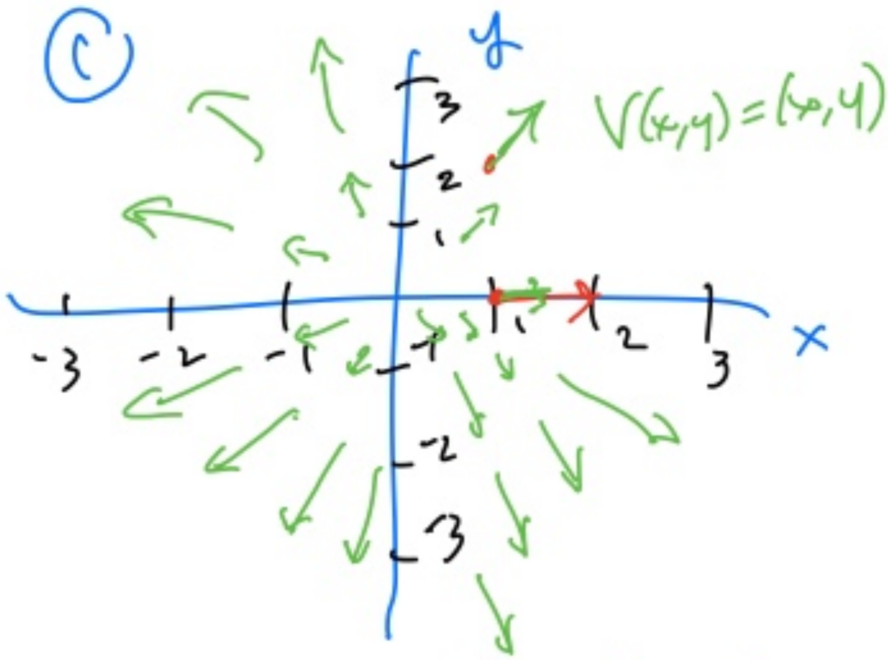
$$= \int_{-1}^{1}\int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(-1+\sqrt{2-x^2-y^2}\right) dy\,dx$$

Graph these vector fields.

① $V(x,y) = (x,y)$

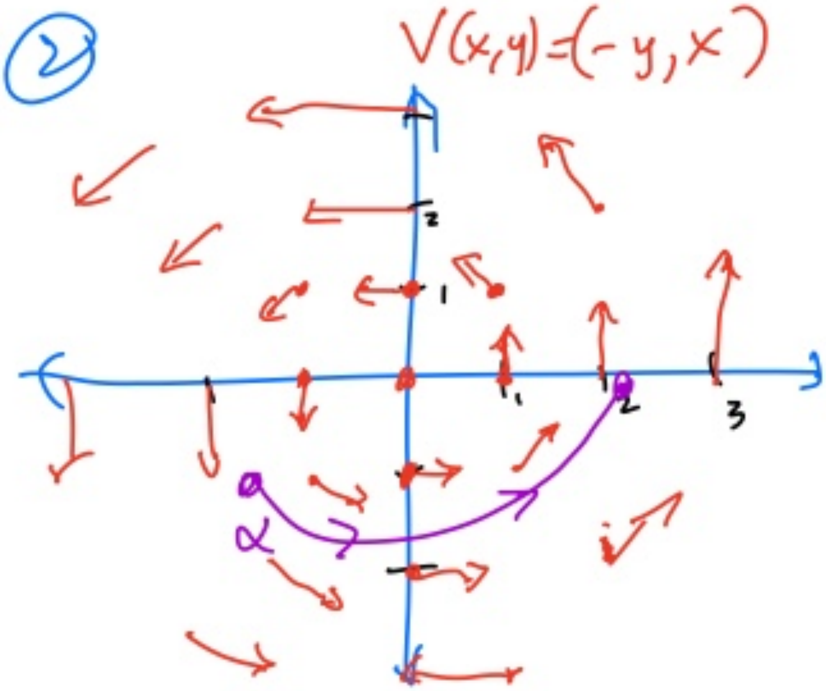
② $V(x,y) = (-y,x)$

①



(x,y)	$V(x,y)=(x,y)$
$(0,0)$	$(0,0)$
$(1,0)$	$(1,0)$
$(-2,7)$	$(-2,7)$

②



(x,y)	$V(x,y)=(-y,x)$
$(0,0)$	$(0,0)$
$(1,0)$	$(0,1)$
$(0,1)$	$(-1,0)$
$(1,1)$	$(-1,1)$
$(0,-1)$	$(1,0)$
$(0,1)$	$(-1,0)$

$\oint_\alpha V\cdot ds$ = positive

if this is a force, $\oint_\alpha \vec{V}\cdot d\vec{s}$ = work done by the force.

Fundamental Theorem of Calculus.

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

$$\int_1^2 x^2\,dx = \int_1^2 \left(\tfrac{1}{3}x^3\right)'\,dx$$

$$= \tfrac{1}{3}(2)^3 - \tfrac{1}{3}(1)^3 = \frac{7}{3}$$

Sometimes we can't use it.

$$\int_1^2 e^{x^2}\,dx = \int_1^2 (\text{aaaaah\ldots})'\,dx$$

Nice situation for line integrals:
Sometimes, our vector field

$$V(x,y,\dots) = \nabla f(x,y,\dots)$$

is the gradient of a function.

Suppose $\alpha(t)$ is a curve

$$\alpha(t) = (x(t), y(t), \dots) \quad a \le t \le b$$

$$\oint_\alpha V \cdot ds = \int_a^b V(\alpha(t)) \cdot \alpha'(t)\,dt$$

$$\Rightarrow \oint_\alpha \nabla f \cdot ds = \int_a^b \underbrace{\nabla f(\alpha(t)) \cdot \alpha'(t)}\, dt$$

$$\frac{d}{dt}\left(f(\alpha(t))\right)$$

$$= \underbrace{f'(\alpha(t))}_{\nabla f(\alpha(t))} \cdot \alpha'(t)$$

$$= \int_a^b \frac{d}{dt}\left(f(\alpha(t))\right) dt$$

$$\underset{\text{FTC}}{=} f(\alpha(b)) - f(\alpha(a)) = f\left(\begin{matrix}\text{end}\\\text{point}\end{matrix}\right) - f\left(\begin{matrix}\text{beg.}\\\text{point}\end{matrix}\right)$$

Example: $V(x,y) = (x,y)$

$$\nabla(f) = (\underset{f_x}{x}, \underset{f_y}{y})$$

Let $f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2$

$$\nabla f = (x,y).$$

Let $\alpha(t) = \left(4\cdot 2^{-t} + t^2, 3 - t^3\right)$

$$-1 \leq t \leq 2$$

$$\oint_\alpha V \cdot ds = \int_{-1}^{2} V\left(4\cdot 2^{-t} + t^2, 3 - t^3\right) \cdot \left(-(\ln 2)\, 4\cdot 2^{-t} + 2t, -3t^2\right) dt$$

$$= \int_{-1}^{2} \nabla f \cdot ds = f(\text{end pt}) - f\left(\substack{\text{beg} \\ \text{pt.}}\right)$$

$$f(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$$

$$\text{end pt} = f(\alpha(2)) = f(4 \cdot 2^{-2} + 4, 3 - 8)$$

$$= f(5, -5)$$

$$= \frac{5^2}{2} + \frac{(-5)^2}{2} = \frac{25}{2} + \frac{25}{2}$$

$$= \boxed{25}.$$

Physics: F is conservative force

if $F = \nabla(\underbrace{-\text{potential}})$